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Sreten Mastilović[1], Branislav Djordjević[2], Aleksandar Sedmak[3], Snežana Kirin[2]*

SIZE EFFECT ASSESSMENT OF K_{Jc} EXPERIMENTAL DATA USING THE TWO-STEP-SCALING METHOD

PROCENA UTICAJA VELIČINE UZORKA NA K_{Jc} EKSPERIMENTALNE REZULTATE PRIMENOM METODE DVOSTEPENOG SKALIRANJA

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Adresa autora / Author's address:
[1)] University of Belgrade, Institute for Multidisciplinary Research, Belgrade, Serbia
email: misko.mastilovic@imsi.bg.ac.rs
[2)] University of Belgrade, Innovation Centre of the Faculty of Mechanical Engineering, Belgrade, Serbia
[3)] University of Belgrade, Faculty of Mechanical Engineering, Belgrade, Serbia

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Ključne reči
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- lomna žilavost

Abstract

The phenomenon of ductile-to-brittle transition (DBT) in ferritic steels, widely used in design of nuclear reactor pressure vessels, has been a pervasive semi-centennial research topic. Due to the extremely pronounced experimental data scatter, the statistical approach to characterization of this problem has become inevitable from the earliest analyses. In the present study, the fracture toughness parameters derived from the EURO fracture toughness dataset for 22NiMoCr37 reactor steel are used with the aim to explore the utility of the recently proposed two-step-scaling method. Two widely different temperatures (-154 °C and -91 °C; belonging to the lower shelf and the DBT transition regions of fracture toughness, in respect) are selected to demonstrate the accuracy of extrapolation and interpolation of the fracture toughness CDF (cumulative distribution function) and the pertinent issues related to the method application. The fracture toughness measure used is the critical value of the stress intensity factor used in the master curve K_{Jc} (MPa√m). The obtained predictions are in good agreement with the experimental results and well within the inherent experimental data scatter. The prediction of the fracture toughness CDF obtained by extrapolation using the novel two-step-scaling method is reasonably conservative.

Izvod

Pojava žilavo-krtog prelaza (DBT) kod feritnih čelika, široko korišćena u nuklearnoj industriji pri projektovanju reaktorskih posuda pod pritiskom, rasprostranjena je tema istraživanja već pola veka. Zbog izuzetno izraženog rasipanja eksperimentalnih rezultata, statistički pristup karakterizaciji ovog problema pokazao se neizbežnim još od najranijih analiza, U ovoj studiji, parametri lomne žilavosti izvedeni su korišćenjem EURO baze podataka o lomnoj žilavosti reaktorskog čelika 22NiMoCr37 sa ciljem utvrđivanja upotrebljivosti nedavno predložene metode dvostepenog skaliranja. Dve veoma različite temperature (-154 °C i -91 °C; koje pripadaju, tim redom, donjem nivou i žilavo-krtom prelaznom režimu lomne žilavosti) odabrane su zarad prikazivanja tačnosti ekstrapolacije i interpolacije funkcije kumulativne raspodele lomne žilavosti i relevantnih pitanja vezanih za primenu metode. Pritom korišćena mera lomne žilavosti, K_{Jc} (MPa√m), je kritična vrednost faktora intenziteta napona koji se koristi u metodi Master krive. Dobijena predviđanja su u saglasnosti sa eksperimentalnim rezultatima i komotno u okviru inherentnog rasipanja podataka. Predviđanje funkcije kumulativne raspodele lomne žilavosti, dobijeno ekstrapolacijom korišćenjem nove metode dvostepenog skaliranja, pokazalo se kao prilično konzervativno.

INTRODUCTION

The occurrence of the DBT (ductile-to-brittle) transition in reactor pressure-vessel steels has been an omnipresent research challenge over the last 50 years. Characterization of this problem within the realm of Fracture Mechanics became inevitable from the pioneering studies based on the LEFM to the application of the EPFM concept. To begin with, it cannot be overemphasized that fracture toughness is not an intrinsic material property. The statistical approach, which emerged in the late 1970s, can still be found today as the basis for interpreting fracture toughness data in DBT transition range. The topic had been practically dormant until the first half of the 20th century and introduction of the weakest-link statistics and development of extreme value theory by Tippett (1902-1985). The Weibull's trailblazing study /1/ laid down foundation of the theory of statistical size effect due to strength randomness. During the last four decades, an extensive theoretical, experimental and computational literature has accumulated. Among numerous statistical studies concerned specifically on the cleavage fracture toughness of ferritic steels that make use of the weakest-link and the Weibull statistics, stand out the empirical approach by Landes and coworkers, the Beremin local model, the Master Curve model of Wallin and others, the Prometey model as outlined (for example, see the recent succinct historical surveys /2, 3/).

The statistical approach to the problem based upon the weakest link Weibull theory is founded on a basic premise that the probability of fracture increases with an increase in the probability of the finding of structural defects in the volume of the tested sample, or the engineering structure. The research prominence has emerged from the engineering necessity of transferability of small-scale test results; 'the need to extrapolate from laboratory tests to much larger real structures, and the recognition that the strength of a brittle and a quasi-brittle structure may be significantly impaired when its size increases beyond the usual range,' /4/.

The focus of the present study is the demonstration of applicability and usefulness of the two-step-scaling (2SS) procedure recently proposed, /5, 6/. This new approach, briefly summarized in the following section, makes no assumptions of any kind about the parameters of the Weibull distribution, but is entirely driven by experiment.

THE 2SS APPROACH TO SIZE EFFECT MODELLING OF K_{Ic} CDF IN THE DBT TRANSITION REGION

The objective of the size-effect investigation, /5, 6/, has been the estimation of the 2-parameter Weibull CDF of the critical value of J-integral (J_c), in the DBT temperature range. Pertinent equations are rewritten here by using CDF ($K_{Jc}\,|\,\beta$, η), that is, the critical value of the stress intensity factor used in the master curve K_{Jc} as the fracture toughness measure. In general case, the Weibull CDF can be rewritten in the form,

$$\mathrm{CDF}(K_{Jc}\,|\,\beta,\eta_*)=1-\exp\left\{-\left(\frac{K_{Jc}W^{\kappa}}{\eta_*+\eta_\infty W^{\kappa}}\right)^{\beta(W)}\right\}, \quad (1)$$

where: β and η_* are Weibull shape and scale parameters, respectively; while W designates the characteristic size (e.g., width or thickness B) of the C(T) specimen. In the present analysis, both Weibull parameters (the slope included) are considered size dependent. Also, only the limit case $\eta_\infty = 0$ is considered since it results in conservative CDF estimates within the 2SS framework (consult the original article /4/ for details). The size-independent Weibull scale parameter η_* in scaled space, CDF($K_{Jc}W^\kappa$) vs. $K_{Jc}W^\kappa$, is defined by the fracture toughness scaling condition,

$$\eta_*=\eta W^{\kappa}=\text{const.}, \quad (\eta_{B*}=\eta B^{\kappa}=\text{const.}), \quad (2)$$

as illustrated in Fig. 1a. (Notably, the analysis can also be defined in terms of non-dimensional size (W/W_0, or B/B_0) normalised using an arbitrarily selected reference state (B_0 = 25 mm ≈ 1 inch being a common choice).

The entire mapping from the original space CDF(K_{Jc}) vs. K_{Jc} to the scaled space, CDF($K_{Jc}W^\kappa$)W^ξ vs. $K_{Jc}W^\kappa$, is defined by a pair of scaling parameters (κ, ξ) as the core of the 2SS method. The former is defined by Eq.(2) while the latter is defined by scaling condition,

$$S_*=SW^{\xi}=\text{const.}, \quad (S_{B*}=SB^{\xi}=\text{const.}), \quad (3)$$

which defines the common CDF slope (S_*) in scaled space, which scales with the probability density function (PDF) maxima of the Weibull distribution (Fig. 1b). (It should be noted that the equality of slopes is imposed only in the scaled space while allowing $\beta = \beta(W)$, in general). The forms of Eq.(2) and Eq.(3) given in parenthesis refer to the Weibull CDF(K_{Jc}), Eq.(1), written in terms of the effective thickness B of the C(T) specimen.

The scaling parameters can be obtained for the available data-set pair by using the following relationships,

$$\kappa=\log_{\left(\frac{W_j}{W_i}\right)}\left(\frac{\eta_i}{\eta_j}\right), \quad \xi=\log_{\left(\frac{W_j}{W_i}\right)}\left[\frac{\Xi(\beta_i)}{\Xi(\beta_j)}\right], \quad (4)$$

where: indices i, j (= 1, 2, …) designate the pair of input experimental datasets corresponding to sample sizes, e.g., W(= W_1, W_2, ... = 25, 50, ...). Scaling parameters defined by Eqs.(4) follow respectively from the constancy conditions expressed by Eqs. (2) and (3) which are the gist of the 2SS method. It cannot be overemphasized that in the case of the size-independent Weibull slope (β = const. ≠ $f(W)$), the scaling parameter $\xi \equiv 0$ by definition, and the two CDFs in Fig. 1a overlap, which makes the second scaling step unnecessary.

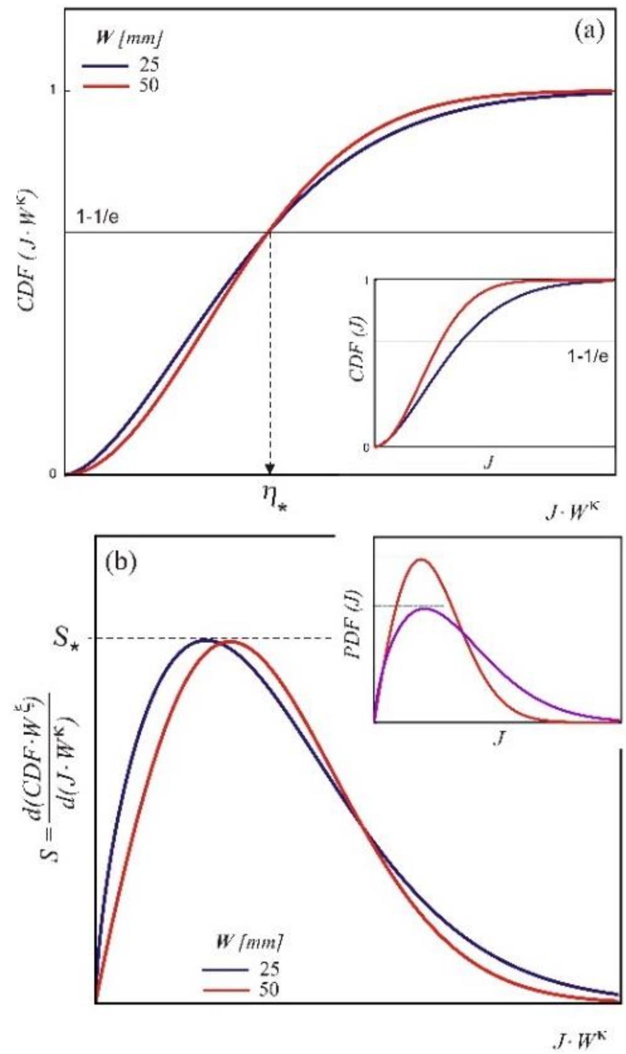

Figure 1. Schemes of two scaling steps defined by: (a) the constancy condition Eq.(2), and corresponding scaling parameter κ Eq.(4); and (b) constancy condition Eq.(3) and scaling parameter ξ Eq.(4).

Finally, once:
- the size-independent Weibull scale parameter η_*, Eq.(2),
- the scaling parameter ξ, Eq.(4),

- and corresponding size-independent CDF slope S_*, Eq.(3), are determined, the value of shape function,

$$\Xi(\beta)=\beta\left(1-\frac{1}{\beta}\right)^{1-\frac{1}{\beta}}\exp\left[-\left(1-\frac{1}{\beta}\right)\right], \quad (5)$$

can be calculated for each particular specimen size W (or B)

$$\Xi(\beta|W,\xi)=S_*\eta_*W^{-\xi}, \quad (\Xi(\beta|B,\xi)=S_{B*}\eta_{B*}B^{-\xi}). \quad (6)$$

As defined by Eq.(5) and illustrated in Fig. 2, the shape function provides 1 : 1 correspondence with the Weibull slope ($\Xi \leftrightarrow \beta$) under the constraint of the sigmoid shaped CDF (the inset of Fig. 2), /7/. It is difficult to overstate that size independency of Weibull parameters η_* and S_*, and the pair of scaling exponents (κ, ξ) makes them universally applicable for all C(T) sizes.

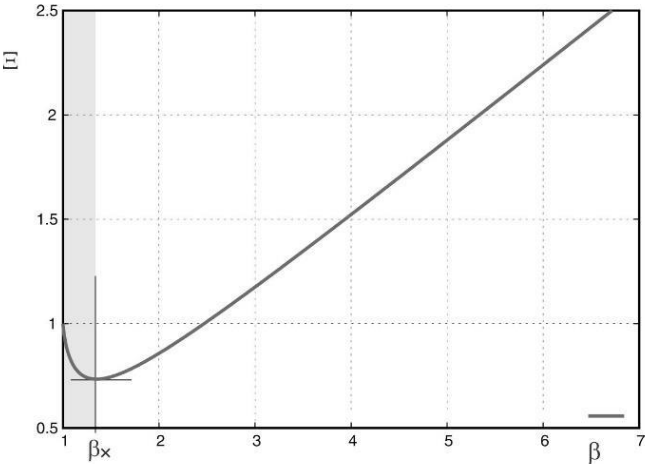

Figure 2. Shape function defined by Eq.(5) vs. Weibull shape parameter β.

Equations (1) through (6) summarize the 2SS method under the limitation of $\eta_\infty = 0$ being a convenient assumption in the case of the constraint of only two experimentally available datasets, which is not so rare when the statistically representative dataset is taken into account. Recall that this assumption ($\eta_\infty = 0$) ensures the conservative estimate of fracture toughness CDF within the framework of the present method. Accordingly, the Weibull CDF can be written as:

$$\mathrm{CDF}(K_{Jc}|\beta,\eta_*)=1-\exp\left\{-\left(\frac{K_{Jc}W^{\kappa}}{\eta_*}\right)^{\beta(W)}\right\}, \quad (7a)$$

that takes into account the effect of the specimen linear dimension (specifically, effective width W). As stressed in /4/, due to the 3-D geometrical similarity of the C(T) specimen ($B \propto W$), it is straightforward to represent the size effect of Eq.(7) in terms of thickness B, if preferable:

$$\mathrm{CDF}(K_{Jc}|\beta,\eta_{B*})=1-\exp\left\{-\left(\frac{K_{Jc}B^{\kappa}}{\eta_{B*}}\right)^{\beta(B)}\right\}. \quad (7b)$$

Finally, for normalised size the preceding CDF expressions take the following forms:

$$\mathrm{CDF}(K_{Jc}|\beta)=1-\exp\left\{-\left(\frac{B}{B_0}\right)^{\kappa\beta}\left(\frac{K_{Jc}}{\eta_0}\right)^{\beta(B)}\right\}=$$

$$=1-\exp\left\{-\left(\frac{W}{W_0}\right)^{\kappa\beta}\left(\frac{K_{Jc}}{\eta_0}\right)^{\beta(W)}\right\}, \quad (7c)$$

where: quantities with subscript '$_0$' correspond to an arbitrarily selected reference state (e.g., 1T C(T) specimen with B_0 = 25 mm ≈ 1 inch is a frequent choice).

PREDICTIONS OF CDF (K_{Jc}) OF DIN 22NiMoCr37 REACTOR STEEL AT LOW TEMPERATURES

Validation of the accuracy of 2SS method predictions of fracture toughness CDF is performed by using the critical value of stress intensity factor used in the master curve (K_{Jc}) reported in /8/. This experimental data is obtained originally from the EURO fracture toughness dataset generated by ten European laboratories in order to provide an experimental database 'sufficiently large to study specimen size and temperature effects on cleavage fracture toughness' in the DBT regime, /9/. The data quantify the fracture behaviour of tempered reactor steel 22NiMoCr37, frequently used in nuclear power plants for pressure vessels /9/. The experimental K_{Jc} data are taken from Annex 2 of /6/, which excludes block SX9 from the complete EURO dataset for which existence of certain macroscopic material variability is confirmed /8, 9/. (Only data points reported as 'valid data' are used.)

Lower shelf region (interpolation case): T = -154 °C

This test temperature belongs to the lower shelf region characterized by practically immediate initiation of multiple cleavage cracks which account for the lack of a significant size effect, /9, 10/. Nonetheless, the very modest size sensitivity is used to demonstrate the 2SS application. Thus, the 2SS analysis starts with data fitting of two input experimental datasets (W = 25, 100 mm), illustrated by triangle symbols in Fig. 3, with 2-parameter Weibull CDF. The fitting results in determination of the Weibull scale and shape parameters (η and β, respectively) given in the first two rows of Table 1. Values corresponding to W = 25, 100 mm are inputs used to calculate the interpolated estimate for W = 50 mm (bold font). (Note that Heerens and Hellmann /9/ conclude that there is no significant size effect at this temperature which is not inconsistent with up to ±10 % difference in Weibull parameters presented in the Table).

Table 1. Weibull scale and shape parameters and corresponding shape function for three different C(T) specimen sizes.

W	η	β	Ξ
25	40.0	6.40	2.385
100	36.0	7.40	2.7485
50	37.95	6.89	2.560

Shape function values Ξ (in the last column of Table 1) are calculated using Eq.(5) once β values are known. With regards to data fitting results (η and β) presented, it should be noted that in the case of highly irregular data, the fitting procedure allows more flexibility and subjectivity then perhaps desirable, which emphasizes the importance of statistically large and high-quality experimental datasets.

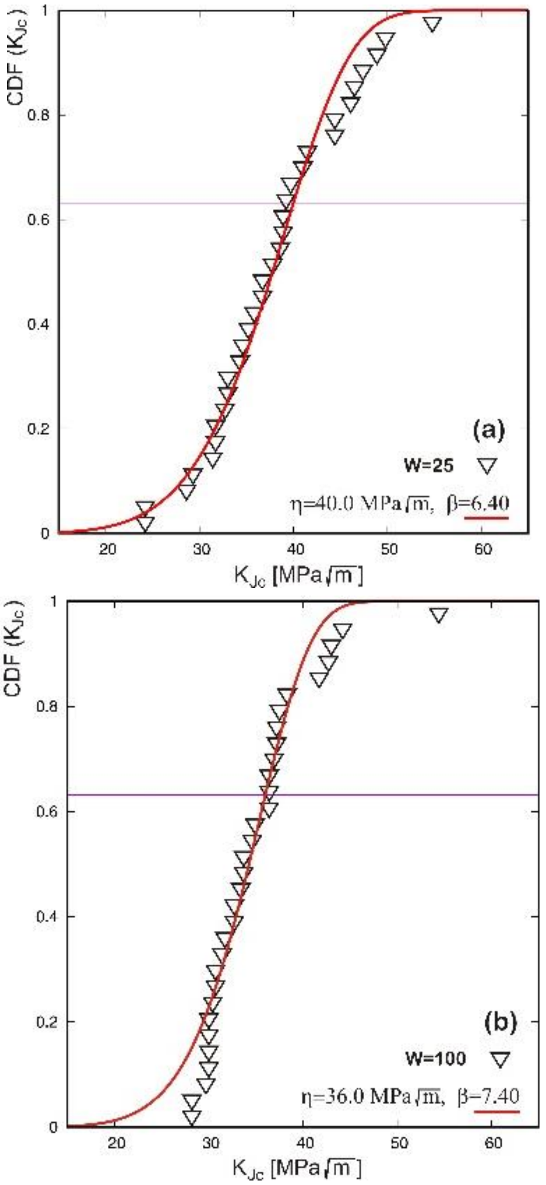

Figure 3. Inputs for the 2SS method ('2 points'): experimentally obtained $K_{Jc(exp)}$ CDF (triangle symbols) for input CT sample sizes $W = 25$, 100 mm and corresponding 2-parameter Weibull distribution fits. CT samples corresponding to: (a) $W = 25$, and (b) $W = 100$ have 32 data points each obtained from two different laboratories (GKSS and Siemens /8/). (Note the data fitting of tails in the case (b) leaves a lot to be desired, but the Weibull slope can be determined with reasonable confidence.)

The pair of scaling parameters (κ, ξ) are calculated from Eq.(4),

$$\kappa = \log_{\left(\frac{100}{25}\right)}\left(\frac{40.0}{36.0}\right) = 0.0760\,,$$

$$\xi = \log_{\left(\frac{100}{25}\right)}\left(\frac{2.385}{2.7485}\right) = -0.1023\,, \qquad (8)$$

for values of input parameters given in Table 1.

The size-independent Weibull scale parameter η_* in scaled space $CDF(K_{Jc}W^{\kappa})$ vs. $K_{Jc}W^{\kappa}$, is calculated using the constancy condition Eq.(2) and CDF values from Table 1:

$$\eta_* = 40.0\cdot 25^{0.0760} = 36.0\cdot 100^{0.0760} = 51.09 = \text{const.} \qquad (9)$$

The common CDF slope (S_*) in scaled space can be calculated based on Eq.(6) for each particular C(T) specimen dimension,

$$S_* = \frac{1}{\eta_*}\Xi(\beta|W)W^{\xi} = \frac{1}{51.09}2.385\cdot 25^{-0.1023} =$$

$$= \frac{1}{51.09}\cdot 2.7485\cdot 100^{-0.1023} = 0.03358 = \text{const.} \qquad (10)$$

The shape function value for C(T) sample $W = 50$ mm can be calculated by using Eq.(6),

$$\Xi(\beta|W = 50, \xi = -0.1023) =$$

$$= 0.03358\cdot 51.09\cdot 50^{-(-0.1023)} = 2.560\,. \qquad (11)$$

Shape function value $\Xi = 2.560$, Eq.(11), uniquely determines the Weibull shape parameter $\beta = 6.89$, based on Fig. 2 (in the domain corresponding to the sigmoid shape of the Weibull CDF /4, 7/), or Eq.(5).

The satisfactory comparison of the prediction in the interpolation domain (red line) and experimental data (circles) is illustrated in Fig. 4. Since the experimental dataset for $W = 50$ mm is of relatively large size in a statistical sense (36 data points), the comparison is rather indicative and suggests that the interpolated CDF is well within the experimental data scatter.

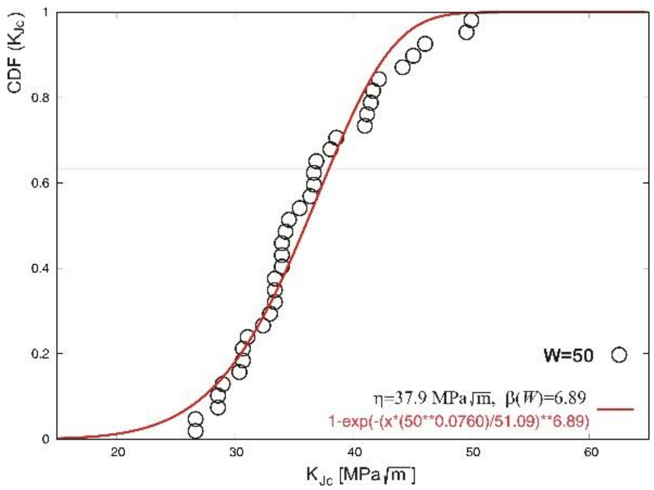

Figure 4. Output of the 2SS procedure: interpolated K_{Jc} CDF for C(T) sample size $W = 50$ mm (line) based on inputs in Fig. 3. The circles correspond to the actual experimental dataset, /8/.

DBT transition region (extrapolation case): T = -91 °C

The application of the 2SS approach starts with data fitting of two input experimental datasets ($W = 25$, 100 mm), illustrated by triangles in Fig. 5, with 2-parameter Weibull CDF. Fitting results in the determination of Weibull scale and shape parameters (η and β, respectively) are given in the first two rows of Table 2. Once again, the values of the shape function Ξ are calculated by using Eq.(5), once β values are known. The pair of scaling parameters (κ, ξ) is calculated from Eq.(4),

$$\kappa = \log_{\left(\frac{100}{25}\right)}\left(\frac{120}{92}\right) = 0.192\,,$$

$$\xi = \log_{\left(\frac{100}{25}\right)}\left(\frac{1.70}{2.60}\right) = -0.307, \qquad (12)$$

for input parameter values given in Table 2.

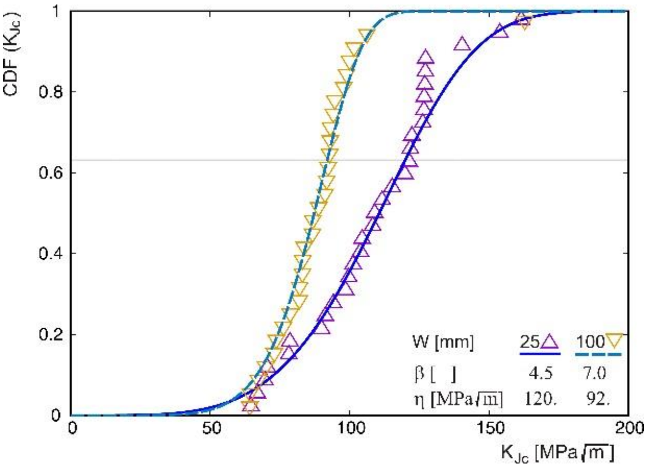

Figure 5. Inputs for the 2SS method ('2 points'): experimentally obtained $K_{Jc(exp)}$ CDF (triangles) for C(T) specimen sizes W = 25, 100 mm and corresponding 2-parameter Weibull distribution fits. The C(T) sample corresponding to: (a) W = 25 mm has 31 data points; while (b) W = 100 mm has 34 data points obtained from three different laboratories, /8/.

Table 2. Weibull scale and shape parameters and corresponding shape function. Values corresponding to W = 25, 100 are inputs used to calculate the extrapolated estimate for W = 200 (in bold).

W	η	B	Ξ
25	120	4.5	1.70
100	92	7.0	2.60
200	80.5	8.65	3.21

The size-independent Weibull scale parameter η_* in scaled space CDF($K_{Jc}W^\kappa$) vs. $K_{Jc}W^\kappa$, is calculated using the constancy condition Eq.(2) and CDF values from Table 1:

$$\eta_* = 120 \cdot 25^{0.192} = 92 \cdot 100^{0.192} = 223 = \text{const.} \qquad (13)$$

The common CDF slope in scaled space can be calculated based on Eq.(6) for each specimen size W,

$$S_* = \frac{1}{\eta_*}\Xi(\beta|W)W^{\xi} = \frac{1}{223}1.70 \cdot 25^{-0.307} =$$
$$= \frac{1}{223}2.60 \cdot 100^{-0.307} = 0.00284 = \text{const.} \qquad (14)$$

The shape function value for C(T) specimen size W = 200 mm can be calculated using Eq.(6) and calculated values of size-independent parameters,

$$\Xi(\beta|W = 200, \xi = -0.307) = 0.00284 \cdot 223 \cdot 200^{-(-0.307)} = 3.21\,. \quad (15)$$

The shape function value Ξ = 3.21 uniquely determines the Weibull shape parameter β = 8.65 based on Fig. 2 (in the domain corresponding to the sigmoid shape of the Weibull CDF /4/), and Eq.(5).

The red solid line in Fig. 6 illustrates the Weibull CDF extrapolation for W = 200 mm and offers the comparison with experimental data (circles), as reported in /8/. These extrapolation results are considered satisfactory and reasonnably conservative, bearing in mind the irregularity of experimental datasets reflecting the inherent stochasticity of fracture toughness in the DBT temperature region.

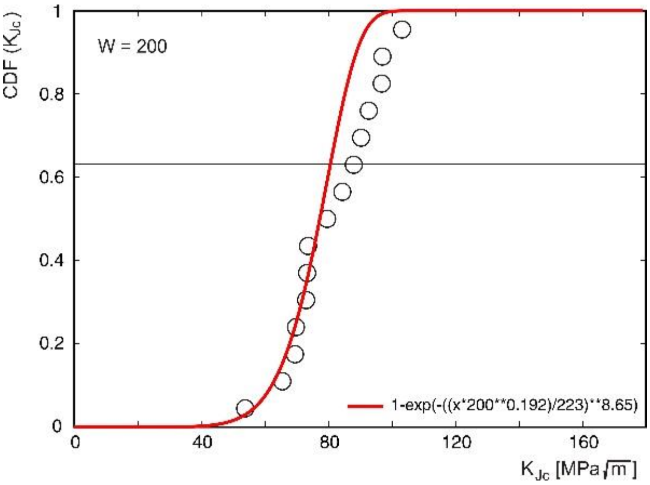

Figure 6. Output of the 2SS method: extrapolated K_{Jc} CDF for C(T) sample size W = 200 mm (line) based on inputs from Fig. 5. The 15 circles correspond to actual experimental dataset obtained from two different laboratories, /8/.

CONCLUSIONS

The presented case study is dedicated to the research of the ability of the novel two-step-scaling approach to estimate fracture toughness CDF of ferritic steels in the DBT range in both interpolation and extrapolation domains of structural sizes. Obtained results suggest that the results of fracture toughness assessment are well within the inherent epistemic uncertainty and random variability of the problem. The two-step-scaling procedure appears as a useful approach to size effect assessment of K_{Jc} results in the DBT temperature region, as well as in the lower shelf region, whenever experimental data follow the basic premise of the weakest link Weibull statistics, that the probability of fracture increases with an increase in the probability of the finding of structural defects in the volume of the specimen/structure.

The validation procedure also highlights the importance of statistical sample size (number of experiments per C(T) specimen size) that will be subject of further investigation. The reliability of fracture toughness estimates is directly conditioned by the reliability of input data reflected by Weibull parameter values, obtained by fitting of the experimental data.

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