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Excitation of guided waves in materials with negative refraction

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Abstract

We study, both analytically and numerically, electromagnetic beam transmission through a layered structure that includes a waveguide slab of metamaterial with negative refraction. The resonant excitation of the leaky guided modes, via attenuated total reflection configuration, can lead to the formation of an anomalous lateral shift in the transmitted beam, as well as to the zero reflection (i.e., high or even total transparency). However, we demonstrate a trade-off between high transparency and high lateral beam shift.

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1. Introduction

Many years ago, electromagnetic wave propagation in an isotropic medium with a negative dielectric permittivity $\varepsilon(\omega) < 0$ and negative magnetic permeability $\mu(\omega) < 0$, over a common band of frequencies ω, has been studied theoretically by Veselago [1]. It was demonstrated that propagation of light, under such conditions, can exhibit very unusual properties: negative refraction, reversed Doppler shift and inverse Cerenkov radiation. Since in such media, the wave vector $\vec{k}$, the electric field $\vec{E}$, and the magnetic field $\vec{H}$ of a wave form a left-handed orthogonal set, they are labeled as left-handed (LH) media, as opposite to the conventional right-handed (RH) media. In addition, there appears a negative group velocity (directed oppositely to the wave phase velocity). No naturally existing LH medium has yet been discovered. Therefore, it became necessary to turn to artificial, man-made materials, called metamaterials. Initially, LH metamaterials (LHM) have been fabricated in the gigahertz range of frequencies [2] and, quite recently, in telecommunication [3] and visible [4,5] range of wavelengths. From the point of view of possible applications, it is worth noting that actual LHM are composed of unit cells of finite dimensions and thus are heterogeneous. In order to consider them as continuous media and introduce an effective dielectric permittivity and effective magnetic permeability that are simultaneously negative, it is necessary to observe the homogenization limit [6]. Nevertheless, the optical properties of such LHM have attracted considerable attention in the past several years in efforts to develop novel optical devices, including perfect lenses [7].

One of the possible applications of LHM, that have not yet been fully explored, is their ability to guide modes along a waveguide (with LHM core and RH cladding) in unusual fashion, as compared to a waveguide that is made completely of conventional RH materials. Although the guiding conditions of a waveguide slab with negative refraction [8,9] and excitation of guided modes [10] have been studied to a degree, more comprehensive investigation of these phenomena is needed. In order to excite guided waves, it is necessary to satisfy phase-matching conditions that can only be achieved with the use of a properly chosen layered structure. In that case, the energy of the guided modes can leak out of the layered structure and thus, lead to their reversible damping. This is in contrast to an irreversible damping that is due to the energy absorption within the layered structure itself. Namely, a reversibility of the leaky guided wave (LGW) damping allows for a resonant energy pumping by an incident beam, that makes possible the perfect matching of the source and the sink

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and leads to zero reflectivity or, when losses can be neglected, to the total transparency of a waveguide slab [11]. The crucial role of a LGW is well established as a mean of the giant lateral reflected beam shifts when a LGW is guided by a single interface between RH and LH materials [12]. A study of the structure and the basic properties of electromagnetic waves guided by a LH waveguide slab show their dramatically different properties from those in a conventional waveguide. For the purpose of this paper, we are interested only in the excitation of the so called "fast" guided modes with the phase velocity higher than the phase velocity in an infinite medium with the same refractive index. This is in contrast to the "slow" guided modes that are out of the scope of this paper. A LH waveguide can support the fast guided modes of both TE and TM polarizations, but the sign of their total energy flux depends on the properties of the waveguide cladding and core. Thus, they can be "forward" propagating (the total energy flux or the group velocity is directed along their wave vector) or "backward" propagating in the opposite case. This is in contrast with a bulk plane wave in LHM, when a group velocity has to be negative (backward). In fact, the total energy flux of a guided mode consists of the two parts: one from the LH core that is always negative and, one from the RH cladding that is always positive. As a consequence, the group velocity will be positive (forward) when the part from the RH cladding is grater than the part from the LH core and, negative (backward) in the opposite case.

2. Formulation of the problem

The excitation of LGW is usually realized via attenuated or frustrated total internal reflection. The so called Otto configuration (prism-air gap-dielectric) has been used to study the excitation of LGW, replacing a dielectric or a metal by a sufficiently thick LH waveguide slab (semi-infinite LHM) [10,12]. In the present paper, however, in order to study the transport of an incoming radiation beam across the layered structure of the finite width, we add the mirror-symmetrical air gap-prism structure on the rear side of the LH waveguide slab (see Fig. 1). We consider a Gaussian beam that has the beam width w and that is obliquely incident from a medium 1 (usually a prism) upon a two-dimensional, three layered structure (media 2, 3 and 4) and can leak through the medium 5 that is the same as a medium 1. The angle of incidence θ_i is defined with respect to the normal to the interface (z-axes) so that the wave vector component along the interface is $k_i = k_p \sin\theta_i$, where $k_p = \omega(\varepsilon_p\mu_p)^{1/2}/c$ represents the wave number within the prisms, and ε_p, μ_p are relative dielectric permittivity and relative magnetic permeability of the prisms. In numerical calculations we will assume the media 2 and 4 (waveguide cladding) to be the air gaps. However, our analytical results are generally valid for $\varepsilon_1\mu_1 \neq 1$, too. Both air gaps have the width a, while we denote the width of the LH core by d (medium 3, characterized by $\varepsilon_2(\omega)$, $\mu_2(\omega)$). If the beam is TE polarized it is convenient to work with the electric field, while for TM polarization one will rather use the magnetic field of the beam. In both cases, the fields are oriented normally to the plane of incidence, i.e., along y-axis, and are consequently continuous across the boundaries between the media. In the present paper, however, we confine ourselves to the case of TE polarization. The results for TM case can be obtained by mutual replacement of the permittivity and the permeability within the gaps and within the LH core. The electric field of the incident beam at the interface $z = 0$ has the form: $E_i(x, z = 0) = \exp(-x^2/2w_x^2 - ik_ix)$, where $w_x = w/\cos(\theta_i)$. The prisms and the gap layers are assumed non-dispersive and non-dissipative, while the LH core (medium 3) is assumed to be a non-dissipative LHM with negative both $\varepsilon_2(\omega)$ and $\mu_2(\omega)$ that have to be frequency dependent and are shown to be of the following form [2]:

$$\varepsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}; \quad \mu_2(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_r^2} \qquad (1)$$

Here, the parameters that have been chosen to fit experimental data of Ref. [2] are: $\omega_p/\omega_r = 2.5$; $F = 0.56$. Since the input beam is incident from an optically dense medium ($\varepsilon_p\mu_p > \varepsilon_1\mu_1$) at an incident angle larger than the angle of total internal reflection, an interface between media 1 and 2 generates a reflected wave and a wave transmitted in the form of evanescent fields that can couple with the evanescent fields of the guided modes supported by the LH waveguide core. These guided waves appear to be eigenmodes of a LH waveguide slab (when the prisms are removed), that satisfy the following dispersion relations:

$$\Delta_m \equiv 1 - \alpha\tan(\kappa_2 d/2 - m\pi/2) = 0; \quad m = 0, 1, 2, \ldots \qquad (2)$$

Here, $\alpha = \mu_1\kappa_2/\mu_2\kappa_1$; $\kappa_1 = \sqrt{\kappa^2 - \varepsilon_1\mu_1}$; $\kappa_2 = \sqrt{\varepsilon_2\mu_2 - \kappa^2}$; $\kappa = k_ic/\omega$; $d\omega/c \rightarrow d$. The modes are symmetric (with respect to the field profile within the core) or even for $m = 0, 2, 4, \ldots$ and anti-symmetric or odd for $m = 1, 3, 5, \ldots$. They are fast (with respect to their phase velocity) when κ_2 is real, and they are slow when κ_2 is imaginary (κ_1 has to be real for a guided wave). At the same time, the modes can be "forward" (with respect to their group velocity v_g) when $v_g > 0$ and they are "backward" when $v_g < 0$. The group velocity of a guided wave is defined as $v_g = P_x/W$, where $P_x = \int_{-\infty}^{\infty} S_x dz$ represents the integrated x-component of the Poynting vector S_x, while $W > 0$ represents the integrated

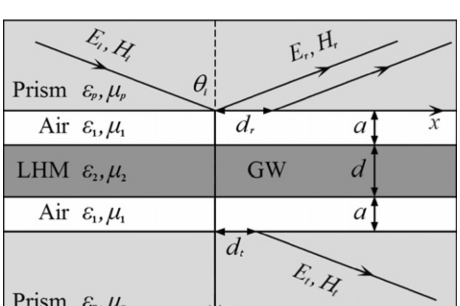

Fig. 1. Geometry of the problem.

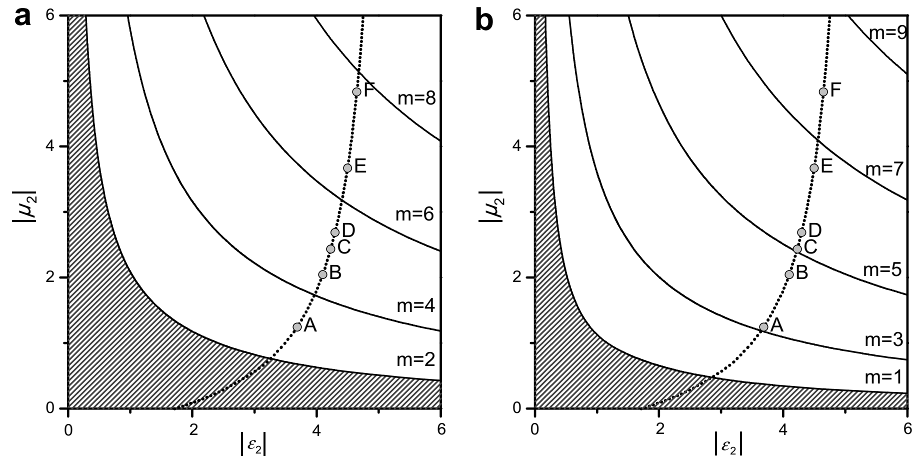

Fig. 2. Existence regions for symmetric $m = 2, 4, 6, \ldots$ (a), and anti-symmetric $m = 1,3,5, \ldots$ (b) modes and $d = 5$. No fast modes exist in the shaded regions. Solid lines represent boundary curves below which modes with the labeled or higher values of m do not exist. Dotted lines represent $|\mu_2(\omega)|$ as a function of $|\varepsilon_2(\omega)|$ according to expressions (1). Points A–F correspond to the plots (a)–(f) in Figs. 3 and 4.

(also with respect to z) energy density of the dispersive media. Thus, the sign of the total energy flux determines the sign of the group velocity, as expected. It follows from (2), that the fundamental ($m = 0$) mode can be slow, but the fast ($m = 0$) mode does not exist at all in a LH waveguide slab. For $m = 1$, the modes can be either slow or fast, while the higher order modes ($m = 2, 3, 4, \ldots$) are obviously fast. Since in the present paper, we are dealing with the fast modes, the lowest order mode to investigate is $m = 1$ anti-symmetric mode. The regions of existence of fast guided modes are presented in Fig. 2a and b, for symmetric ($m = 2, 4, \ldots$) and anti-symmetric ($m = 1, 3, \ldots$) modes, respectively. No fast modes exist in the shaded regions. The solid lines represent boundary curves for each particular m, below which the modes with the labeled and the higher values of m do not exist. Let us mention that $\varepsilon_2(\omega)$ and $\mu_2(\omega)$ are both negative in the frequency range $\omega_r < \omega < \omega_r/(1 - F)^{1/2}$, and this is imposed for the existence of the guided waves if $\varepsilon_1 = \mu_1 = 1$, i.e., when gaps are chosen to be the air. Then, for the parameter values given above $1.75 < |\varepsilon_2(\omega)| < 5.25$ and $|\mu_2(\omega)| > 0$. Dotted lines represent $|\mu_2(\omega)|$ as a function of $|\varepsilon_2(\omega)|$ in the above mentioned range of frequencies. Notice that the frequency decreases when going from the point A towards the point F.

3. Results and discussion

When the prisms are included, the fast LGW can be excited if the angle of incidence θ_i satisfies the following relation $\varepsilon_1\mu_1/\varepsilon_p\mu_p < \sin^2\theta_i < \varepsilon_2\mu_2/\varepsilon_p\mu_p$. For an incident plane wave, zero reflection $|R|^2 = 0$ (or total transparency $|T|^2 = 1$ when losses are negligible) can be achieved if the following condition is satisfied:

$$\sinh[2\kappa_1 a]\cos[\kappa_2 d] - \frac{1-\alpha^2}{2\alpha}\cosh[2\kappa_1 a]\sin[\kappa_2 d] = \frac{\beta_p^2 - 1}{\beta_p^2 + 1}\,\frac{1+\alpha^2}{2\alpha}\sin[\kappa_2 d] \qquad (3)$$

Here, $\beta_p = \mu_1\kappa_p/\mu_p\kappa_1$; $\kappa_p = \sqrt{\varepsilon_p\mu_p - \kappa^2}$. It is worth noting that when the losses are taken into account, ε_2 and μ_2 become complex, and both real and imaginary parts of Eq. (3) have to be simultaneously equal to zero in order to achieve zero reflectivity (see Ref. [12]). Of course, in that case, the total transparency cannot be achieved. Nevertheless, the transparency may remain high when the condition (3) is fulfilled. However, within the plane wave approximation, no lateral shifts in the reflected and the transmitted waves can be observed. If we consider an input beam rather than an incident plane wave, an anomalous lateral shift in a transmitted wave can appear. In fact, there is a trade-off between the effects of high transparency and high lateral shift. As an incident beam narrows, the transparency peak declines from unity while the lateral shift becomes more and more pronounced. The shift can be in forward or backward direction, depending on the gap and the slab widths, as well as on the frequency of the incident radiation. We have solved the problem both analytically and numerically. The following expression describes transmitted electric field distribution of the transmitted beam (the amplitude of the input beam is assumed to be equal to unity):

$$E_t = R_{12}\sqrt{\frac{\pi}{2}}\mid\delta_m\mid w_x \exp\left(-\frac{x^2}{2w_x^2} + \mathrm{i}k_i x + \gamma_m^2\right)]\mathrm{erfc}(\sigma_m\gamma_m);\quad m = 1, 2, 3, \ldots \qquad (4)$$

where R_{12} is the reflection coefficient from a single boundary between media 1 and 2 ($|R_{12}|^2 = 1$); $\delta_m = [4\beta_p/(1+\beta_p^2)]\exp(-2\kappa_1 a)(\partial\Delta_m/\partial\kappa)^{-1}_{\Delta_m=0}$ and is inversely proportional to the integrated energy flux (carried by the excited fast guided mode) over the gaps and LH slab; $\gamma_m = \frac{1}{\sqrt{2}}\left[\delta_m w_x - \frac{x}{w_x} + \mathrm{i}w_x(k_i - k_{m0})\right]$; $\sigma_m = \mathrm{sgn}(\delta_m) = \mathrm{sgn}(v_{gm})$, where v_{gm} is the group velocity of the m mode; $a = \omega a/c$ is the normalized width of the gaps; k_{m0} is the solution of the dispersion Eq. (2) with respect to k_i, and $\mathrm{erfc}(\gamma_m)$ is the complementary error function of the real argument when $k_i = k_{m0}$. One can see, that the sign of v_{gm} determines whether the shift of the transmitted beam will be in the forward or backward direction with respect to the obliquely incident electromagnetic beam. The expression (4) is in excellent agreement with our numerical calculations that

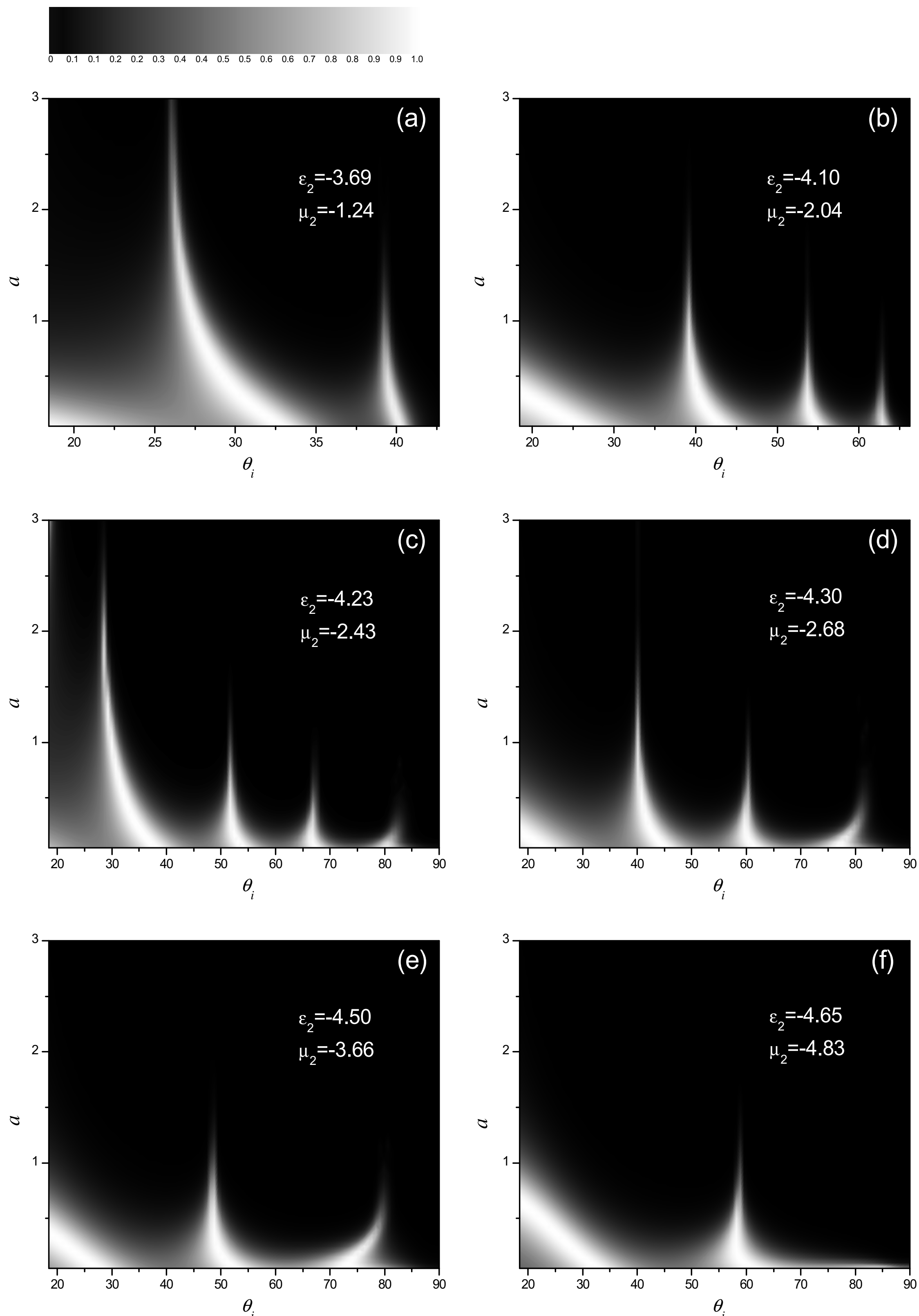

Fig. 3. Contour plots of the transparency as a function of the angle of incidence θ_i (degrees) and the normalized gap width a. Here,$d = 5$, $\varepsilon_p = 10$, $\mu_p = 1$. Plots (a)–(f) correspond to the points A–F in Fig. 2, and different modes for: (a) $m = 2, 3$; (b) $m = 2, 3, 4$; (c) $m = 2, 3, 4, 5$; (d) $m = 3, 4, 5$; (e) $m = 5, 6$ and (f) $m = 7$. In all plots, the higher peaks correspond to the higher m.

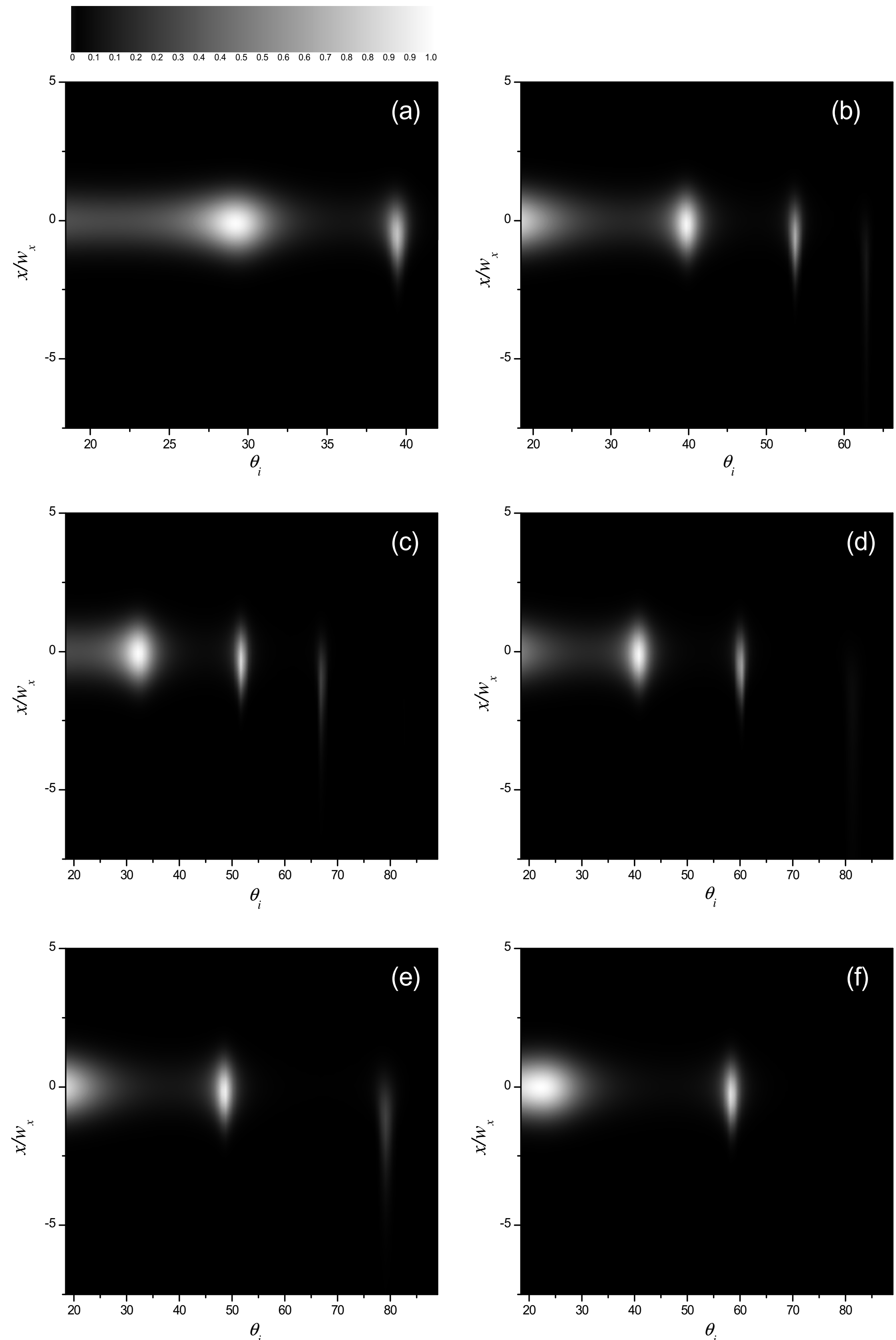

Fig. 4. Contour profiles of the transmitted beams as a function of incident angle θ_i for gap thickness $a = 0.5$. The other parameters have the same values as in the corresponding plots in Fig. 3.

are presented in Figs. 3 and 4. In Fig. 3, we present the contour plots of the transparency $|E_t|^2$ as a function of the angle of incidence θ_i and the gap width a ($d = 5$, $\varepsilon_p = 10$ and $\mu_p = 1$) and various frequencies (plots (a)–(f)) that correspond to the points A–F in Fig. 2. As can be seen, when a increases, the intensity of transmitted light decreases, but the positions of the transparency maxima become more close to the resonant angles of incidence for LGW excitation, i.e., for$k_i = k_{m0}$. In Fig. 4, we present the contour plots of the transparency $|E_t|^2$ as a function of the normalized lateral coordinate x/w_x and the angle of incidence θ_i ($a = 0.5$ and the other parameters are the same as in Fig. 3). Since a is much smaller than d in these numerical examples, lateral beam shifts are always in the negative direction. The trade-off between high transparency and anomalously high lateral shift is clearly seen in all the cases (a)–(f).

4. Conclusions

We have investigated electromagnetic beam transmission through a layered structure that contains metamaterials with negative refraction. The excitation of fast guided modes within a LHM slab, by using the attenuated total reflection configuration, has been studied in detail. The effect of total transparency in the plane wave approximation has been extended to a beam, and the trade-off between high transparency and anomalously high lateral shift has been demonstrated, both analytically and numerically. However, to complete the insight in the phenomena, it is necessary to take into account losses within LHM, as well as to study the excitation of slow guided modes.

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